Effects of Turbulence on a Weakly Ionized Plasma Column

JOSEF SHWARTZ* AND YITZHAK LAVIE†
Technion—Israel Institute of Technology, Haifa, Israel

A theoretical model for predicting the effects of turbulent flow on properties of the positive column in a glow discharge is presented. The model assumes that the ions which are imbedded in the turbulent neutral gas flow acquire the motion of, and are completely correlated with, the neutral gas. Available models for a turbulent pipe-flow and for the positive column of a glow discharge, are then combined to derive the effect of turbulence on ambipolar diffusion, rate of ionization, electric field, electron temperature, and charge distribution. The model predicts that an increase in the Reynolds number of the flow will increase the ionization rate, the electron temperature, and the electric field, and will flatten out the charge-concentration profiles in agreement with past observations and with our own experimental results.

I. Introduction

IN recent years there has been an increased interest in the properties of glow discharges, particularly as related to high-power fast-flow electric-discharge CO_2 lasers. The primary purpose of flowing the gas through the CO_2 lasers is to convect the heat out of the laser cavity, thereby allowing both larger electrical power input and optical power output. It has been known for some time, however, that besides its pure thermal effect, the flow can also change the intrinsic properties of an electric discharge, particularly when the flow is turbulent. This has been observed, for example, by Gentle et al,[1] who measured the effect of gas flow on electric field in the positive column of an argon glow discharge. They observed a sharp upward break in the electric field as the flow Reynolds number approached a value of 1800, independent of discharge current and direction of flow relative to the anode-cathode arrangement. Similar observations were made by Cottingham and Buchsbaum[2] in their investigation of the effect of turbulence on the RF breakdown in gases, and by Granatstein and Buchsbaum,[3] who also used an argon d.c. glow discharge.

These observations have been pursued in considerably greater detail by Garosi et al.[4] They investigated the effects of turbulence on the positive column of an argon discharge, measuring the variation of electric field, electron temperature, ionization rate, and charge distribution with flow rate and Reynolds number. They found, for example, that as the Reynolds number of the flow is increased from zero to 6300, the rate of ionization increased by more than one order of magnitude, the electric field nearly doubled, the electron temperature increased by roughly 10%, the plasma column expanded to fill the tube, and the charge concentration profiles expanded and flattened out.

If the effects listed above are also to be found in mixtures of atomic and molecular gases used in electric lasers, they could contribute significantly to the performance of these lasers. Thus, an expansion of the plasma column under turbulent flow conditions could improve temperature distribution and gain in the lasing gas, and might also favorably affect the stability of the discharge. The dependence of electron temperature on Reynolds number in a turbulent flow discharge may provide a means for controlling and optimizing, at least to some extent, the electron energies in electric-discharge lasers.

The possibility of using the gas flow, which is essential to the thermal control of the high-power discharge, to improve the stability of the discharge is indeed intriguing. Hill[5] and Shirahata and Fujisawa,[6] for example, produced high-power d.c. discharges in an initially supersonic flow of a H_e, N_2, and CO_2 mixture. The mixture was then shocked and reduced to a subsonic flow, immediately past the entrance to the discharge region. This resulted in marked improvements in discharge stability, permissible power loading and power output,[5] and saturation parameter.[6] Eckbreth and Owen[7] found that by controlling the velocity profile in a discharge and by increasing the level of turbulence, they were able to enhance the stability of the discharge and increase the electrical power input. Biblarz and Nelson[8] have similarly observed that when a discharge is struck in atmospheric air, the introduction of grid-generated turbulence will markedly improve current distribution and power deposition in a diffuse (pre-arcing) discharge. All these effects are at best only qualitatively understood, and there is clearly a need for a better understanding of flow and discharge interactions. Such understanding could contribute significantly to improvements in electric discharge laser design, and may be applicable in other areas as well.

In the present paper we focus our attention on the effects of a fully developed turbulent pipeflow on the properties of a diffuse glow discharge in a gas under moderate pressure, and we present a model which can predict the flow and discharge interactions for this case. The model is based on the assumption that the charged particles (ions) in the discharge acquire the turbulent motions of the neutral background gas and are fully correlated with the neutral gas. This was shown to be the case in practical turbulent-flow discharges,[4,9] except in the case where the discharge was so constricted that its cross-section was much smaller than that of the neutral gas flow.[10] Our interest, motivated by possible applications to electric lasers, is concentrated on cases where the discharge is diffused so that it occupies the bulk of the flow cross-section. Having thus related the plasma turbulence to the neutral gas turbulence, we then combine a simple model of turbulence with either a classical or more advanced model for the positive column of a glow discharge in order to obtain solutions which show how the properties of the discharge are affected by the level of turbulence in the discharge (Secs. II and III).

The solutions obtained from this model are then compared with the experimental results of Garosi et al.[4] The theoretical results for the increase of the electric field, electron temperature, and ionization rate with turbulence are found to be, respectively, within 10%, 30%, and a factor of two of the experimental results.

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* Senior Lecturer, Department of Aeronautical Engineering. Member AIAA.

† Graduate Student, Department of Physics.

The model is also compared with our own preliminary experimental results for a highly diffused glow discharge in turbulently flowing argon gas, and generally good agreement is obtained (Sec. IV).

II. Ambipolar Diffusion in Turbulent Pipe Flow

In the light of the results obtained by Granatstein[9] and Garosi et al.,[4] it will be assumed here that the presence of charged particles in a weakly ionized plasma, with charge particle concentrations typically in the range of $10^{12}/\text{cm}^3$, does not affect the turbulence characteristics of the neutral host gas. On the other hand, it will be assumed that the turbulent motion of the ions is completely correlated with that of the neutral gas.

This was indeed shown to be the case by both Granatstein[9] and Garosi et al.,[4] who, by measuring charge fluctuations, examined the turbulence spectra of a weakly ionized turbulent plasma flow, and compared the results with known spectra of neutral gas flow. The two spectra were shown to be nearly identical, indicating a close correlation between the motion of the neutrals and the ions. The only difference detected was that in the plasma the inertial subrange appeared to be smaller than in the neutral gas flow.

As a consequence of this correlation, it will also be assumed that the ion mobility (or diffusivity) can be linearly superimposed on the turbulent fluctuations of the neutral gas, to obtain an effective diffusivity. The effective ion diffusion coefficient in a turbulent flow, D_+^*, can therefore be taken as the sum of the intrinsic ion diffusion coefficient, D_+ (such as measured in a stationary gas), and the eddy diffusivity in the turbulent flow, ε, namely

$$D_+^* = D_+ + \varepsilon \tag{1}$$

The diffusion coefficient for the ion-electron pairs in a neutral plasma is given by[11]

$$D_a = (D_+\mu_e + D_e\mu)/(\mu_+ + \mu_e) \tag{2}$$

where μ_+ and μ_e are the ion and electron mobilities and D_+ and D_e are the ion and electron diffusion coefficients. D_a is the ambipolar diffusion coefficient, and it represents some balance between the fast-moving and highly diffusive electrons and the slower ions. For conditions where the electron energies are much higher than those of the neutral atoms or ions ($T_e \gg T_g$), it can be shown[11] that Eq. (2) reduces to

$$D_a = D_+(T_e/T_g) \tag{3}$$

where T_e is the electron temperature and T_g is the neutral gas temperature. In electric discharges this is the condition usually encountered.

Turbulence has an immediate effect on the ion diffusion coefficient, as we have seen above, and thus it will also affect the ambipolar diffusion coefficient. On the basis of Eq. (3), the effective ambipolar diffusion coefficient in a turbulent flow D_a^* should be written as

$$D_a^* = D_+^*(T_e/T_g) = (D_+ + \varepsilon)(T_e/T_g) \tag{4}$$

or

$$D_a^*/D_a = 1 + (v/D_+)(\varepsilon/v) = 1 + Sc(\varepsilon/v) \tag{5}$$

where v is the kinematic diffusion coefficient of the neutral host gas, and Sc is the Schmidt number for the specific ion in its host gas.

Several models can be used for determining ε/v for turbulent flow in a tube as a function of the Reynolds number Re and of the radial position in the tube r. Combining Reichardt's equation[12] for the velocity distribution across the tube and the Blasius equation[12] for the friction coefficient, the following equation is obtained for the eddy diffusivity:

$$\frac{\varepsilon}{v} = \left(\frac{0.079}{2}\right)^{1/2} \frac{Re^{7/8}}{30}\left[1-\left(\frac{r}{R}\right)^2\right]\left[1+2\left(\frac{r}{R}\right)^2\right] \tag{6}$$

where R is the tube radius. This equation for ε/v was used throughout the present work, both because of its simplicity and because of its relative accuracy in describing all the ranges in a turbulent pipe flow. Deissler's model[12] for a turbulent pipe-flow was tried too, and the results obtained for the turbulence and discharge interactions were essentially the same as those obtained with Eq. (6).

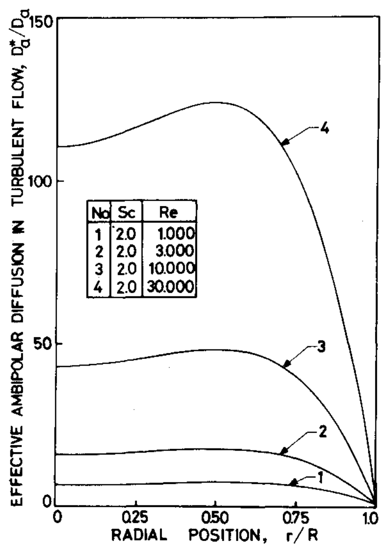

Fig. 1 Enhancement of ambipolar diffusion coefficient in a turbulent flow.

The effective ambipolar diffusion coefficient in a turbulent plasma flowing through a tube, as a function of radial position and with Re and Sc as parameters, can now be computed from Eqs. (5) and (6). One such example is shown in Fig. 1. There the variation of D_a^* with the radius (with Re as a parameter) is shown, for a case where gas pressure and temperature, and therefore also the Schmidt number, can be taken as uniform and constant throughout the tube. As seen in Fig. 1, the enhancement of ambipolar diffusion in turbulent flow very closely resembles the enhancement of momentum transfer, or effective diffusivity, in such flows. As we shall see later, this enhancement of ambipolar diffusion by turbulence is the key to the turbulence effects on the microscopic and gross properties of the positive column in a glow discharge.

III. Effects of Turbulence on the Positive Column

For the purpose of demonstrating the anticipated effects of turbulence on the positive column of a glow discharge, let us first consider the classical positive column, as described by Schottky's theory.[13] This theory is valid for a wall-stabilized gas discharge, in which the loss by diffusion of ion-electron pairs to the walls is exactly balanced by the rate of ion-electron production by ionization. The governing equation in this case is the charge conservation equation

$$\nabla(D_a^*\nabla n) + zn = 0 \tag{7}$$

where n is the electron or ion concentration, and z is the rate of ionization. The effective ambipolar diffusion coefficient D_a^* was used here instead of D_a, to include possible effects of turbulence on charge conservation.

Making the usual assumption that there is no variation of charge concentration in the direction of the tube axis, one has

$$(1/r)(d/dr)[rD_a^*(dn/dr)] + zn = 0 \tag{8}$$

or, upon normalization,

$$(1/\bar{r})(d/d\bar{r})[\bar{r}\bar{D}(d\bar{n}/d\bar{r})] + (zR^2/D_a)\bar{n} = 0 \tag{9}$$

where we have defined

$$\bar{r} = r/R, \qquad \bar{D} = D_a^*/D_a, \qquad \bar{n} = n/n(0) \tag{10}$$

$n(0)$ being the charged particle concentration on the tube axis. The boundary conditions for Eq. (9) require that $\bar{n} = 0$ at $\bar{r} = 1$, and, from symmetry considerations, that $d\bar{n}/d\bar{r} = 0$ at $\bar{r} = 0$.

In line with the classical theory for the positive column, we will assume that gas pressure and temperature, as well as

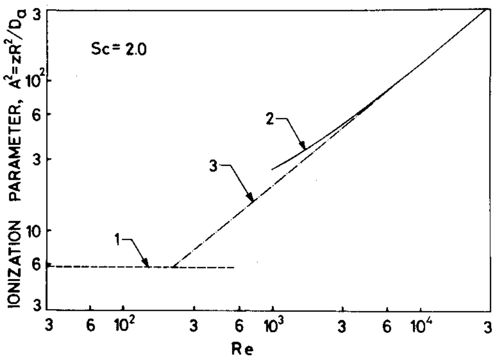

Fig. 2 Ionization parameter (or eigenvalue) vs Reynolds number for 1) Schottky's theory; 2) Schottky's theory in turbulent flow; 3) Asymptotic solution for large Re [see Eq. (13)].

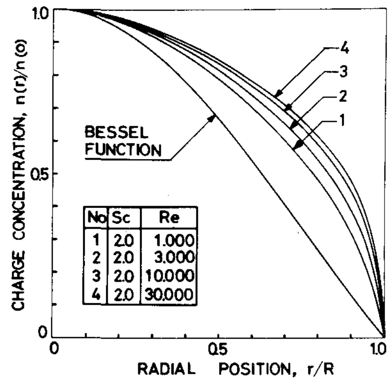

Fig. 3 Charge concentration profiles in turbulent flow discharges.

electron temperature, are uniform and constant throughout the column, and consequently D_a and Sc are constant.

Equation (9) and its boundary conditions define an eigenvalue problem. In the classical case, with $\bar{D} = 1$, the solution is the zeroth order Bessel function

$$\bar{n} = J_o[R(z/D_a)^{1/2}\bar{r}] \tag{11}$$

and the eigenvalue is

$$A = R(z/D_a)^{1/2} = 2.405 \tag{12}$$

as is well known. According to the model presented here, $\bar{D}$ is a function of both Re and Sc, as well as of the radius $\bar{r}$, for a positive column in turbulent flow. The problem is still an eigenvalue problem, but different eigenvalues will now be found for different combinations of Re and Sc, so that the eigenvalue is therefore a continually varying function of Re and Sc; $A = A(Re, Sc)$. Equation (9), combined with Eqs. (5) and (6), and subject to the boundary conditions $\bar{n} = 0$ at $\bar{r} = 1$ and $d\bar{n}/d\bar{r} = 0$ at $\bar{r} = 0$, was solved numerically, with Re and Sc as parameters.

The results for $A = A(Re, Sc)$ are shown in Fig. 2. Note that these results can be interpreted as the growth in the ionization rate z required to balance the increased loss of charged particles to the walls which is caused by increased turbulent diffusivity. By examining a range of solutions for A vs Re and Sc, it was found that a close curve-fit for the numerical results of $A(Re, Sc)$ is given by the expression

$$A^2 = (2.405)^2 + 0.08Re^{4/5}Sc^{7/8} \tag{13}$$

as can be seen in Fig. 2 for one particular value of Sc.

Another direct product of the numerical solution is the charge concentration profiles $\bar{n}(\bar{r})$ with Re and Sc as parameters. Some typical results are shown in Fig. 3. Increasing the level of turbulence Re at constant Sc, tends to flatten out the charge concentration profiles, as would be expected. This would also indicate a more uniform current distribution and more uniform heating across the tube with increasing Re. This result could be significant for various discharge devices, particularly electric lasers, on two counts: first, the dispersion of the electrical current would allow an increase in the electrical power input for a gas-temperature limited device; and second, it might contribute to an increased stability of the discharge.

Finally, the effect of turbulence on electron energy and electric field can be derived directly from the results obtained for the eigenvalue A (or the ionization frequency z). In the classical model (Schottky's theory), where single step ionization and Maxwellian electron energy distributions are assumed, the relation between the rate of ionization and electron temperature is given by[13]

$$z = ap\frac{m}{e}\frac{2}{(\pi)^{1/2}}\left(\frac{2kT_e}{m}\right)^{3/2}\frac{eV_i}{kT_e}\exp\left(-\frac{eV_i}{kT_e}\right) \tag{14}$$

where a is a constant, p is the gas pressure, and V_i the ionization potential of the gas. When z is expressed in terms of the eigenvalue A, and the proper constants are introduced, the equation for the electron temperature becomes

$$\left[\left(\frac{eV_i}{kT_e}\right)^{1/2}\right]^{-1}\exp\left(\frac{eV_i}{kT_e}\right), p = \frac{6.7\times 10^7}{A^2}(CpR)^2 \tag{15}$$

where C is a constant characteristic of the gas considered.

In a stationary gas or in a laminar flow discharge, under conditions corresponding to Schottky's theory, the eigenvalue A is fixed at 2.405. The electron temperature is then a function of (pR) only. When the gas flow becomes turbulent, with all other conditions remaining unchanged, A becomes a function of Re and Sc, and so does T_e according to Eq. (15); namely $T_e = T_e(Re, Sc, pR)$.

Some results depicting the variation of T_e/V_i with (CpR), and with Re as a parameter, are shown in Fig. 4. For a given (CpR) an increase of the level of turbulence causes an increase in the electron temperature. At the lower (CpR) values, the growth of T_e with Re can be quite significant. This result should have been anticipated, since we showed earlier that increasing Re causes a substantial increase in the rate of ionization z and the only way this could happen would be through an increase of the electron energy or temperature.

This change of electron temperature with Re may be of some significance in the design of electric discharge lasers, since it allows T_e to be increased at a given (pR) by means of increasing the level of turbulence in the gas. It also introduces a way of maintaining a fixed T_e value while increasing (pR), by appropriately changing the level of turbulence in the gas. Finally, when T_e is known, the electric field of a positive column can be computed, using an energy balance equation for electrons, such as[13]

$$E = \frac{2.4}{R}\left(\frac{V_i}{\varepsilon}\frac{kT_e}{e}\frac{\mu_+}{\mu_e}\right)^{1/2} \tag{16}$$

where ε is the fraction of the electron energy lost in an ionizing collision.

We have thus demonstrated how a turbulent flow will interact with an electric discharge and change the properties of the positive column. In the example used here a very simple model of the positive column (Schottky's theory) was used. More elaborate and refined models of the positive column could be used, if the discharge conditions so required. However, the

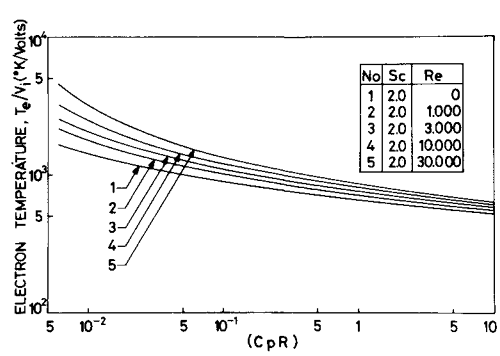

Fig. 4 Electron temperature variations in turbulent flow discharges.

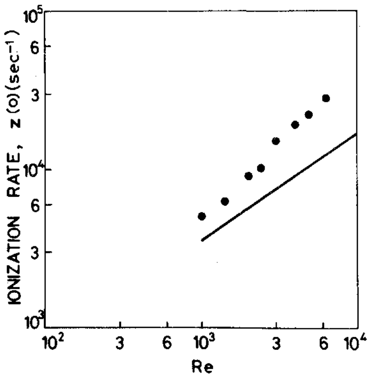

Fig. 5 Ionization rate on discharge tube axis as a function of Reynolds number: 1) as deduced from rate of plasma decay in afterglow[4] (points); 2) as predicted by elementary theory, Eq. (13) (straight line).

nature of the turbulence discharge interactions and the major effects predicted by the model which was outlined here will remain essentially the same, as long as diffusion plays some significant role in stabilizing the discharge. To verify this theoretical model, it should be compared with experimental observations and results. Such comparisons are presented in Sec. IV.

IV. Comparison with Experimental Results

Garosi et al[4] conducted an extensive study of the effects of turbulent gas flow on the properties of the positive column in an argon discharge. Their experimental setup consisted of a tube 2.2 cm in diameter and 76 cm long, through which argon was made to flow at 25 torr pressure and at progressively increasing rates, from about 0.020 *lps* (STP), corresponding to a slow laminar flow, to nearly 2 *lps* (STP), corresponding to a turbulent flow with a Reynolds number in excess of 6000. A DC discharge was struck between a hot, oxide-coated cathode and a cold, needle-shaped anode inserted in the tube, on its axis. The discharge current used was 1.8 A. Langmuir-type electrostatic probes and thermocouples were used to analyse the properties of the positive column.

Garosi et al[4] measured the effects of flow rate and level of turbulence (Re) on gas temperature, electron temperature, charge concentration, and electric field. By measuring the rate of decay of plasma density in the afterglow region, downstream from the anode, they also obtained an indirect measurement of the rate of ionization in the positive column, at the center of the tube, $z(0)$.

A striking feature of the afterglow, as reported,[4] is that beyond a Reynolds number of about 1000, the length of the afterglow does not change with flow velocity, indicating that the rate of plasma decay, beyond $Re = 1000$, is proportional to the flow velocity, or Re. Consequently, they found that the plasma loss rate and the rate of ionization in the positive column is roughly proportional to Re once the turbulence becomes pronounced and self-sustained. Since ambipolar diffusion was found to be the dominant plasma loss mechanism in these experiments,[4] the observations cited above seem to support our model for the effects of turbulence on the positive column, and particularly on the ionization rate (see Fig. 2).

Using the basic model for turbulent-flow and discharge interactions presented in Secs. II and III, we can obtain a rough estimate of the ionization rates for the experiments of Garosi et al.[4] For a first estimate we can neglect the deviations of their experimental conditions from the classical model and use the approximate relation, Eq. (13), for the dependence of the ionization parameter (zR^2/D_a) on the Reynolds number. For the conditions reported by Garosi et al., the average Schmidt number is 3.8 and the average value of the ambipolar diffusion coefficient is 93 cm^2/sec. Introducing these values into Eq. (13), with $R = 1.1$ cm, z can be computed as a function of Re. A comparison between the measured ionization rates and those computed from this model is shown in Fig. 5. As seen, the model accurately predicts the trend of the growth of the ionization rate with Re; a 4/5 power law according to the model as compared to a nearly linear growth found in the experiments. Even the absolute value of the ionization rate is predicted to within a factor of 2.

For a more detailed comparison of the results predicted by our model and the experimental results of Garosi et al., the model must first be modified, since the conditions of the experiments do not conform to the classical theory for the positive column. Particularly the relation for the ionization rate, Eq. (14), should be modified to allow for deviations from single step ionization and Maxwellian electron energy distribution assumed in the derivation of that relation.

Wojaczek[14] carried out an extensive investigation of the properties of argon discharges with no gas flow. Using his results one can derive a relation for the ionization rate in an argon discharge as a function of electron density, electron temperature, and gas pressure and temperature. For the experimental conditions of Garosi et al., this expression is given by

$$z = 0.34n^{2/3}p_o^{1/3}T_e^{-5/6}\exp(-11.5/T_e - KB) \qquad (17)$$

where n is the electron density measured in units of cm^{-3}, p_o is the reduced pressure ($p_o = 273p/T_g$), measured in torr, and T_e is measured in eV. B is a function of the gas pressure, gas and electron temperatures, and charge concentration, and is given by

$$B = 1.5 \times 10^{13}(p_o T_e^2/n). \qquad (18)$$

K is a function of electron temperature and is given approximately by $K \simeq 0.035/T_e^3$. Equation (17) is valid for the range of $2 < B < 30$. The product (KB), as compared to $11.5/T_e$, is a measure of the extent of the deviation from a Maxwellian electron energy distribution; for the experiments of Garosi et al. it is typical of the order of $0.2/T_e$, while B is roughly 5.6.

Garosi et al. showed that for their experimental conditions, ambipolar diffusion is still the dominant charged-particle loss mechanism. Consequently, the basic conservation equation of our model, Eq. (9), still holds, except that the variation of z with n and T_e (and therefore also with position) according to Eq. (17), must be incorporated into Eq. (9). The ambipolar diffusion coefficient for the conditions of Garosi et al., based on measurements made by Chanin and Biodni,[15] is given by

$$D_a = 28/p_o(T_g/0.024)^{2/3}(1 + T_e/T_g) \qquad (19)$$

where T_e and T_g are measured in eV, p_o in torr and D_a in cm^2/sec.

Both the electron and the gas temperature are expected to vary across the tube, the latter more than the former. Garosi et al. measured the change of T_e with r and found relatively small variations. For the gas temperature only results measured at the center of the discharge tube were reported. It can be argued that at the higher flow rates used, convection is the dominant heat transport mechanism and therefore the gas temperature could be nearly uniform across the tube. To simplify the analysis, we will assume that both the electron and gas temperatures do not change across the tube at a given position along the tube.

For the conditions listed above, the charge conservation equation can be written as

$$\frac{1}{\bar{r}}\frac{d}{d\bar{r}}\left(\bar{r}\bar{D}\frac{d\bar{n}}{d\bar{r}}\right) + \left[\frac{z(0)R^2}{D_a}\right]\bar{n}^{5/3}\exp\left[-KB(0)\left(\frac{1-\bar{n}}{\bar{n}}\right)\right] = 0 \qquad (20)$$

where the notation (0) is used to represent the value of a variable at the center of the discharge. This equation is subject to the same boundary conditions as Eq. (9), namely, $\bar{n} = 0$ at $\bar{r} = 1$ and $d\bar{n}/d\bar{r} = 0$ at $\bar{r} = 0$. $\bar{D}$ can still be computed from Eqs. (5) and (6). Equation (20) cannot be solved in closed form since the unknown values of $n(0)$ and T_e are needed for computing $KB(0)$. An iterative solution will therefore be required.

For the purpose of comparing this solution with the experimental results of Garosi et al. we used their measured

Table 1 Plasma properties in a turbulent flow argon discharge; $R = 1.1$ cm, $p = 25$ torr and $I = 1.8$ A[4]

Test conditions				Measured properties			Theoretically predicted properties		
Re	T_g (°K)	Sc	$n(0) \times 10^{-13}$ (cm^{-3})	T_e (eV)	E (V/cm)	$z(0) \times 10^{-3}$ (sec^{-1})	T_e (eV)	E (V/cm)	$z(0) \times 10^{-3}$ (sec^{-1})
1000	603	4.00	4.26	1.04	1.05	5.0	0.930	0.80	3.62
1900	553	3.91	4.26	1.06	1.25	9.0	0.969	0.97	5.41
2400	533	3.87	3.84	1.07	1.35	10.2	0.981	1.03	6.24
3100	513	3.82	3.67	1.09	1.49	15.0	1.000	1.11	7.41
4000	488	3.77	3.53	1.11	1.59	19.0	1.017	1.22	8.7
4800	468	3.72	3.36	1.12	1.68	22.0	1.033	1.33	9.84
6300	443	3.67	3.16	1.13	1.74	28.3	1.057	1.48	11.88

values of $n(0)$. The iterative solution then proceeds as follows: 1) guess a value of T_e; 2) compute $KB(0)$ and D_a; 3) solve Eq. (20) and compute $z(0)$; 4) using Eq. (17) compute T_e from $z(0)$. This has to be repeated until the values of T_e converge. Once the electron temperature is known, the rate of ionization and the electric field E can be computed as well.

In Table 1 the experimental results of Garosi et al. are compared with our model and calculations. As is shown, the model predicts with reasonably good accuracy both the absolute values of the discharge properties and their variations with the Reynolds number of the flow. The largest discrepancy found is between the measured and predicted rates of ionization. It should be remembered, however, that the rate of ionization was measured in a rather indirect fashion (i.e., from the rate of plasma decay past the active region of the discharge), and it is nearly impossible to determine the accuracy of such measurements.

We conducted our own experimental investigation on the effects of turbulence on the properties of the positive column of an argon discharge. The purpose of these experiments was to further validate the previous model and to extend the experimental results to a range typical for CO_2 laser operation, namely pressures between 10 and 40 torr and current densities between 20 and 40 mA/cm^2 with corresponding charge concentrations of $3\text{–}6 \times 10^{11}$ cm^{-3}.

For this purpose we used a discharge tube of 1.9 cm in diameter and about 40 cm long. The cathode was shaped like a nozzle of 1 cm i.d. and the flow was forced through this nozzle into the discharge. This was done so as to disperse the discharge and decrease the tendency to form narrow discharge filaments (or streamers).[16] The anode was a simple copper cylinder. Low-current discharges were used, 50–100 mA. The pressure was varied between 10 and 40 torr, and the flow rates were varied so as to obtain a range of Reynolds numbers between 200 and 7300. The electric field and electron temperature in the positive column were measured by electrostatic probes, and the gas temperature by thermocouples. Some typical results are presented in Tables 2 and 3. These results also exhibit the characteristic growth of the electric field and electron temperature with increased Reynolds number.

No detailed theoretical solutions have yet been obtained for these experiments. The corresponding value of B is in excess of 1000. Wojaczek[14] does offer an equation for the rate of ionization z for the range of $30 < B < 1000$. This expression is given approximately by

$$z = 4.6 \times 10^{-5} n T_e^{-3/2} \exp\left(-\frac{11.5}{T_e} - KB\right) \tag{21}$$

with K and B as defined earlier. When this expression was used in conjunction with the conservation equation for our experimental conditions, unreasonably high eigenvalues and electron temperatures were obtained, indicating that a direct extrapolation to cases with $B > 1000$ is not permissible.

Even though a complete theoretical solution is not possible at this time, one can still predict the *relative* increase of discharge properties, such as T_e and E, with Reynolds number. Let us assume that whatever the proper expression for the ionization rate, the dominant term is of the form $\exp(-b/T_e)$, as in Eqs. (17) and (21), where b depends on discharge condition such as p, n, and Tg, but not on T_e. Let us further assume that the major effects of turbulence on a discharge are properly described by the elementary theory presented in Secs. II and III and Eq. (13), as long as the discharge is controlled by diffusion. Combining these two assumptions we can compute the relative growth of the ionization rate and electron temperature in the following way

$$\frac{\exp(-b/T_e)}{\exp(-b/T_{e1})} \approx \frac{z}{z_1} \approx \frac{A^2}{A_1^2} = \frac{(2.405)^2 + 0.08 Re^{4/5} Sc^{7/8}}{(2.405)^2 + 0.08 Re_1^{4/5} Sc_1^{7/8}} \tag{22}$$

Once the electron temperature is known, the relative change of the electric field can be computed using the relation $T_e \sim (E/P_o)^{2/5}$ as suggested by Garosi et al.[4]

Taking the first measured values of T_e and E in Tables 2 and 3 as known, the subsequent values, corresponding to larger Re values, can be found using Eq. (22). These computed values are included in the last two columns of these tables. For b

Table 2 Plasma properties in turbulent flow argon discharge; $R = 0.95$ cm, $p = 10$ torr, and $I = 100$ mA

Test conditions			Measured properties		Predicted properties	
Re	T_g (°K)	Sc	T_e (eV)	E (V/cm)	T_e (eV)	E (V/cm)
950	332	3.386	2.23	3.09	...	...
1450	325	3.367	2.27	3.18	2.30	3.33
1980	318	3.348	2.30	3.33	2.37	3.59
3130	309	3.322	2.45	3.89	2.47	3.98

Table 3 Plasma properties in turbulent flow argon discharge; $R = 0.95$ cm, $p = 20$ torr, and $I = 50$ mA

Test conditions			Measured properties		Predicted properties	
Re	T_g (°K)	Sc	T_e (eV)	E (V/cm)	T_e (eV)	E (V/cm)
950	332	3.386	2.44	4.67	...	...
1460	324	3.364	2.48	4.95	2.49	4.91
2250	319	3.350	2.52	5.07	2.53	5.11
3560	311	3.327	2.57	5.32	2.59	5.42
4650	307	3.318	2.60	5.65	2.62	5.57
5300	304	3.309	2.64	6.00	2.64	5.68
5920	303	3.305	2.66	6.65	2.66	5.80

we used the expression suggested by Wojaczek,[14] namely $(11.5+KBT_e)$, which yields $b = 20$ for the conditions of Table 1 (with $n \approx 5 \times 10^{11}$), and $b = 40$ for the conditions of Table 2 (with $n \approx 3 \times 10^{11}$).

This last comparison shows that the electric field and electron temperature in a turbulent discharge can be fairly well predicted on the basis of an elementary model for which Eq. (13) was derived. Hence this equation is a useful tool for predicting the effects of turbulence on diffusion controlled discharges.

V. Conclusions

All the changes of the plasma properties in a positive column, observed to be caused by turbulent flow, can be explained in terms of the increase in charge diffusivity caused by turbulence. Combining a simple model for turbulent pipe-flow with existing models for the positive column, the dependence of discharge properties on the level of turbulence can be predicted. Measurements made in turbulent flow argon discharges, in the range of 10–25 torr, generally confirm these predictions.

The model and scheme presented here for analysing the turbulent-flow and discharge interaction problem is quite general and can be applied to various gases and various discharge configurations and conditions. The effects of turbulence are expected to be strongest under those conditions where charge diffusion is a significant loss mechanism. It should be remembered, however, that turbulence can increase the charge diffusivity by several orders of magnitude, so even discharges which are normally dominated by volume recombination can be affected by turbulence if the Reynolds number is sufficiently high.

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